

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

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### Pearson Edexcel International GCSE

Time 2 hours

Paper  
reference

**4PM1/02**

## Further Pure Mathematics PAPER 2



Calculators may be used.

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  - *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.  
Anything you write on the formulae page will gain NO credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  - *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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Pearson

## International GCSE in Further Pure Mathematics Formulae sheet

### Mensuration

**Surface area of sphere** =  $4\pi r^2$

**Curved surface area of cone** =  $\pi r \times \text{slant height}$

**Volume of sphere** =  $\frac{4}{3}\pi r^3$

### Series

#### Arithmetic series

Sum to  $n$  terms,  $S_n = \frac{n}{2}[2a + (n - 1)d]$

#### Geometric series

Sum to  $n$  terms,  $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity,  $S_\infty = \frac{a}{1 - r}$   $|r| < 1$

#### Binomial series

$(1 + x)^n = 1 + nx + \frac{n(n - 1)}{2!}x^2 + \dots + \frac{n(n - 1)\dots(n - r + 1)}{r!}x^r + \dots$  for  $|x| < 1, n \in \mathbb{Q}$

### Calculus

#### Quotient rule (differentiation)

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

### Trigonometry

#### Cosine rule

In triangle  $ABC$ :  $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

### Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

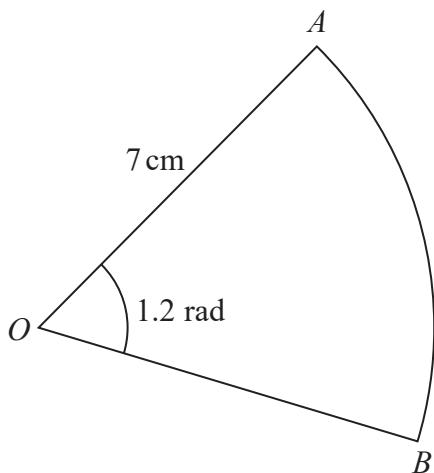


**Answer all TEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1**



**Diagram NOT  
accurately drawn**

**Figure 1**

Figure 1 shows sector  $AOB$  of a circle with centre  $O$  and radius 7 cm.

The angle  $AOB$  is 1.2 radians.

Calculate

(a) the area of sector  $AOB$  (2)

(b) the perimeter of sector  $AOB$  (2)

**(Total for Question 1 is 4 marks)**



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**2** Solve the equation

$$\sin(2\theta - 20)^\circ - \sqrt{3} \cos(2\theta - 20)^\circ = 0 \quad \text{for } 0 \leq \theta \leq 180 \quad (5)$$

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## **Question 2 continued**

**(Total for Question 2 is 5 marks)**



- 3 The curve  $C$  has equation  $y = 9 - x^2$

Use algebraic integration to find the area of the finite region bounded by  $C$  and the  $x$ -axis.

(5)

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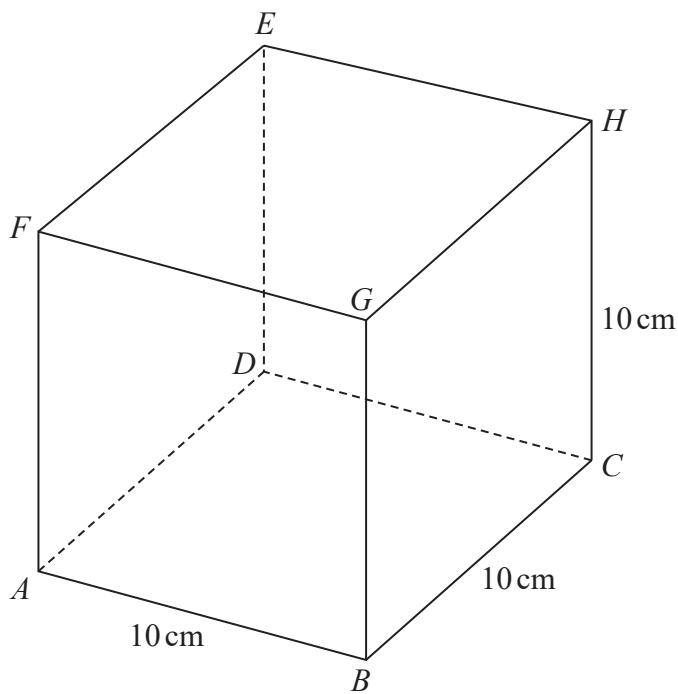


### **Question 3 continued**

(Total for Question 3 is 5 marks)



Diagram **NOT**  
accurately drawn



**Figure 2**

Figure 2 shows a solid cube  $ABCDEFGH$  with sides of length 10 cm.

- (a) Find, in cm, the exact length of  $FC$  (2)
- (b) Find, in degrees to one decimal place, the size of  $\angle FCA$  (2)
- (c) Find, to the nearest degree, the size of the obtuse angle between the plane  $BFH$  and the plane  $BHC$  (5)



## **Question 4 continued**



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## **Question 4 continued**

(Total for Question 4 is 9 marks)



- 5 A particle  $P$  is moving along the  $x$ -axis. At time  $t$  seconds ( $t \geq 0$ ) the velocity,  $v$  m/s, of  $P$  is given by  $v = 3t^2 - 23t + 30$

(a) Find the values of  $t$  when  $P$  is instantaneously at rest.

(3)

At time  $t$  seconds the acceleration of  $P$  is  $a$  m/s $^2$

(b) Find the range of values of  $t$  for which  $a > 0$

(2)

When  $t = 0$ ,  $P$  is at the point with coordinates  $(d, 0)$

Given that, when  $t = 8$ ,  $P$  is at the point with coordinates  $(26, 0)$

(c) find the value of  $d$

(4)



### **Question 5 continued**



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**Question 5 continued**

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## **Question 5 continued**

(Total for Question 5 is 9 marks)



6 The sum of the first  $n$  terms of an arithmetic series is  $S_n$  where  $S_n = n(3 + 2n)$

(a) Find the value of  $S_{20}$

(2)

Given that  $S_n = \sum_{r=1}^n (Ar + B)$

(b) find the value of  $A$  and the value of  $B$

(6)

A different arithmetic series has first term 7 and common difference 4

The sum of the first  $n$  terms of this series is  $T_n$

(c) Use algebra to find the value of  $n$  for which  $T_n = S_n + 252$

(5)



## **Question 6 continued**



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## **Question 6 continued**

(Total for Question 6 is 13 marks)



7

$$f(x) = 2x^2 + px + q \quad \text{where } p \text{ and } q \text{ are integers}$$
$$g(x) = 14x^2 + 37x + 14$$

The equation  $f(x) = 0$  has roots  $\alpha$  and  $\beta$

The equation  $g(x) = 0$  has roots  $\frac{\alpha}{\beta}$  and  $\frac{\beta}{\alpha}$

Given that  $p + q = -4$  where  $p > 0$  and without solving the equation  $g(x) = 0$

(a) find

(i) the value of  $p$

(ii) the value of  $q$

(9)

Given also that  $\alpha > \beta$

(b) show that  $\alpha^2 - \beta^2 = -\frac{3\sqrt{65}}{4}$

(4)



## **Question 7 continued**



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### **Question 7 continued**

(Total for Question 7 is 13 marks)



- 8 (a) Given that  $x$  is measured in radians, complete the table of values for

$$y = \sin x - 3 \cos 2x - 0.5$$

Give your answers to one decimal place.

$x$	0	0.5	0.8	1	1.6	2	2.5	3
$y$			0.3	1.6			-0.8	-3.2

(2)

- (b) On the grid opposite, draw the graph of  $y = \sin x - 3 \cos 2x - 0.5$  for  $0 \leq x \leq 3$

(2)

- (c) Use a formula from page 2 to show that  $\cos 2A = 1 - 2 \sin^2 A$

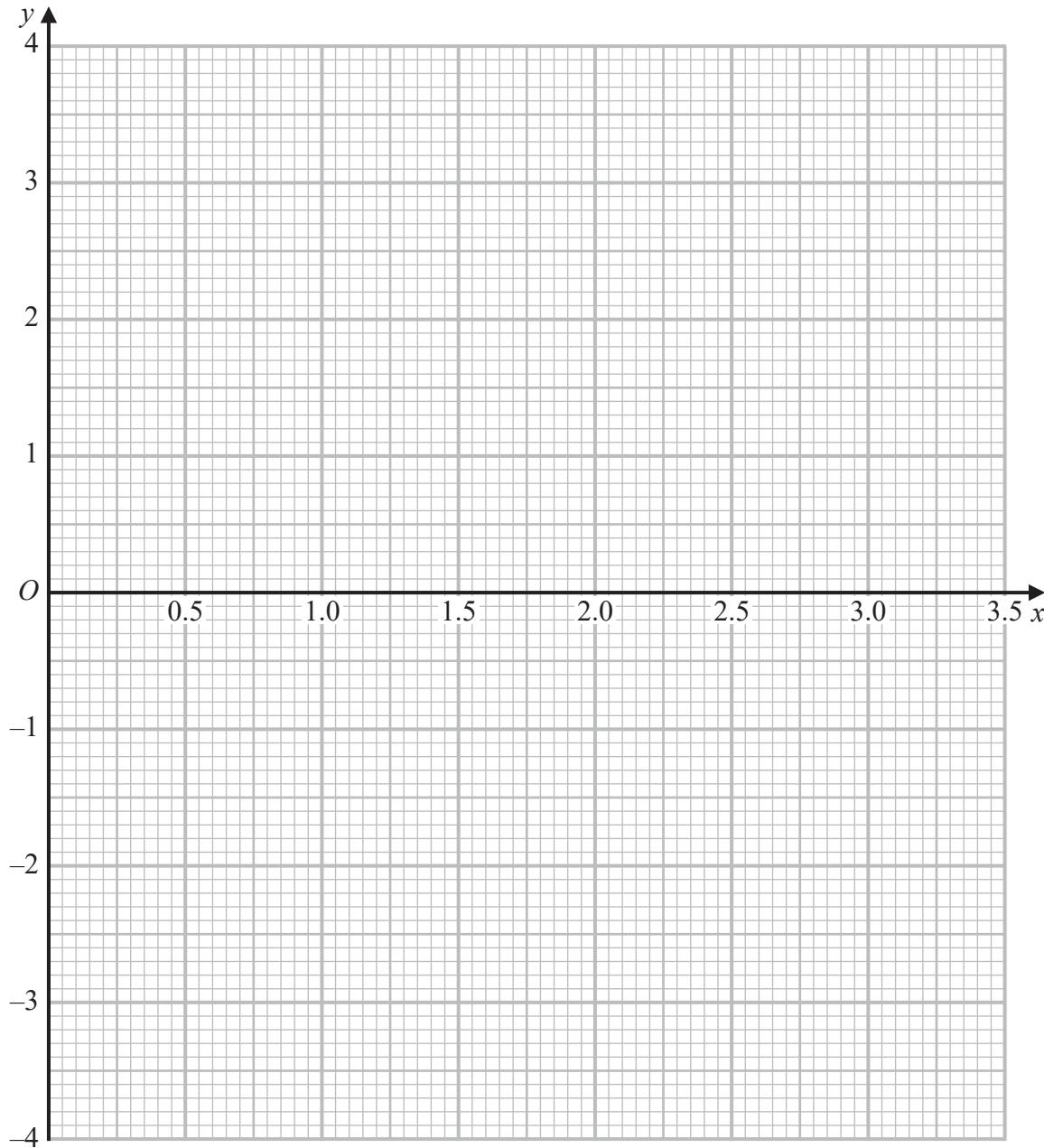
(2)

$$f(x) = 2 \sin x + 12 \sin^2 x - x - 5$$

- (d) By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation  $f(x) = 0$  in the interval  $0 \leq x \leq 3$

(5)



**Question 8 continued**

Turn over for a spare grid if you need to redraw your graph.



**Question 8 continued**

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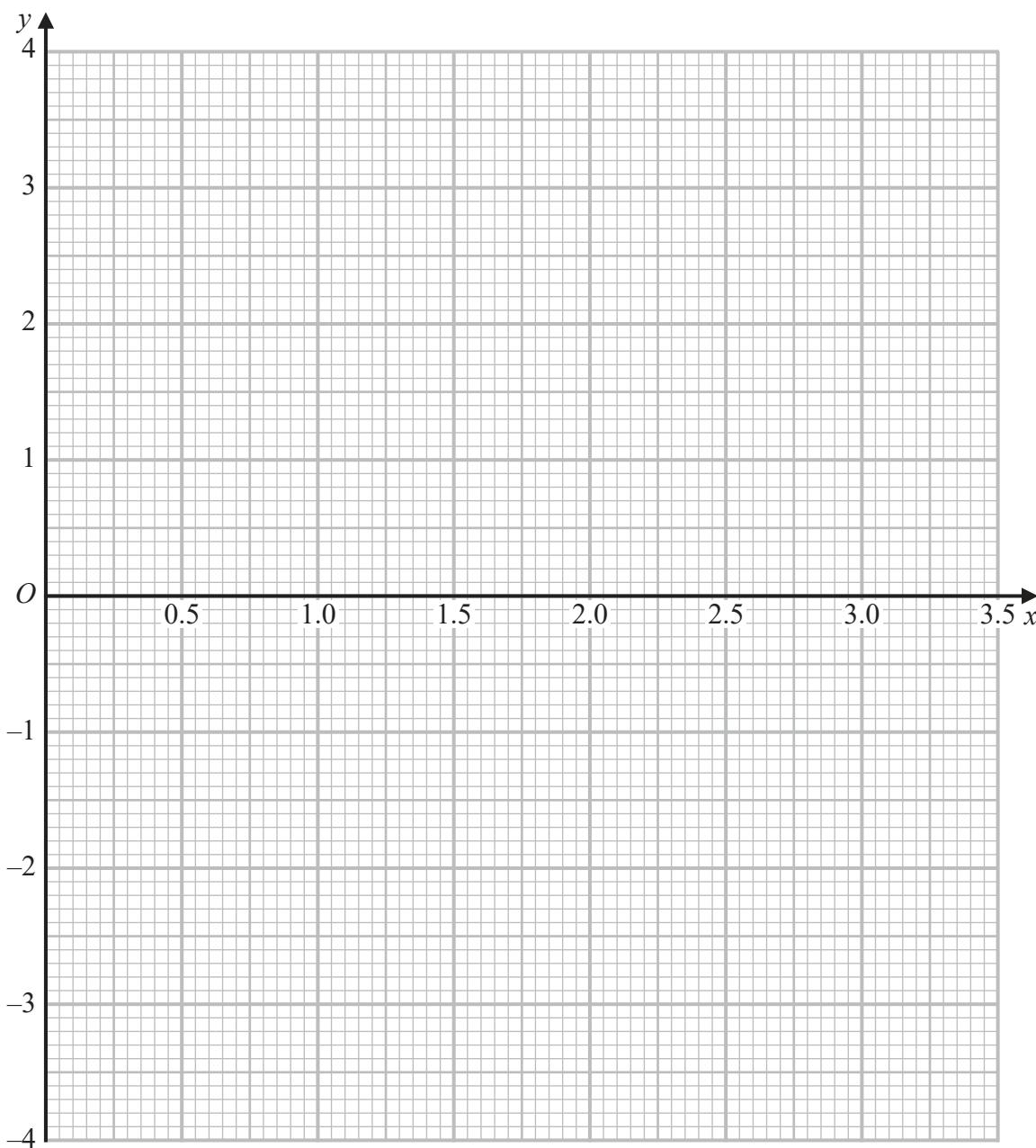
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**Question 8 continued**

**Only use this grid if you need to redraw your graph.**



**(Total for Question 8 is 11 marks)**



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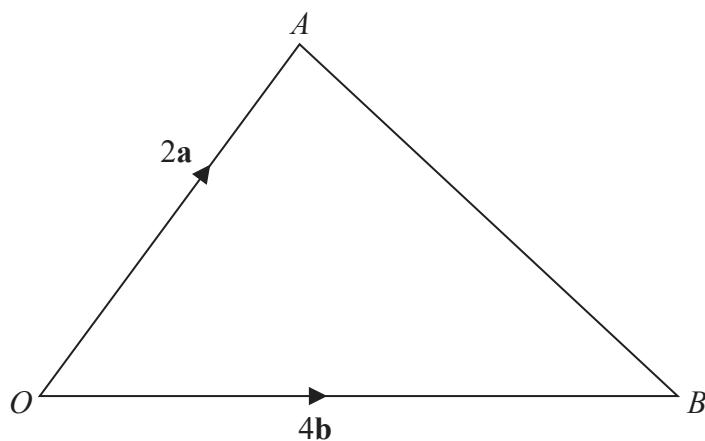


Diagram NOT  
accurately drawn

**Figure 3**

Figure 3 shows the triangle  $OAB$  with

$$\overrightarrow{OA} = 2\mathbf{a} \text{ and } \overrightarrow{OB} = 4\mathbf{b}$$

- (a) Find  $\overrightarrow{AB}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$  (2)

The point  $P$  is the midpoint of  $AB$

- (b) Find  $\overrightarrow{OP}$  as a simplified expression in terms of  $\mathbf{a}$  and  $\mathbf{b}$  (2)

The point  $Q$  lies on  $OP$  such that  $OQ : QP = 3 : 1$

- (c) Find  $\overrightarrow{AQ}$  as a simplified expression in terms of  $\mathbf{a}$  and  $\mathbf{b}$  (3)

The point  $R$  lies on  $OB$  such that  $AQR$  is a straight line.

- (d) Find in its simplest form the ratio  $OR : RB$  (6)



## **Question 9 continued**



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**Question 9 continued**

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### **Question 9 continued**

**(Total for Question 9 is 13 marks)**



10 The curve  $C$  has equation  $y = \frac{2x - 1}{x + 4}$   $x \neq -4$

(a) Write down an equation of the asymptote to  $C$  that is

- (i) parallel to the  $x$ -axis,
- (ii) parallel to the  $y$ -axis.

(2)

(b) Find the coordinates of the points of intersection of  $C$  with the coordinate axes.

(2)

(c) Using the axes on the opposite page, sketch  $C$ , showing clearly the asymptotes and the coordinates of the points of intersection of  $C$  with the coordinate axes.

(3)

The line with equation  $y = x + k_1$  is the tangent to  $C$  at the point  $P$

The line with equation  $y = x + k_2$  is the tangent to  $C$  at the point  $Q$

Given that the  $x$  coordinate of  $P$  is greater than the  $x$  coordinate of  $Q$

(d) using calculus, find the coordinates of

- (i)  $P$
- (ii)  $Q$

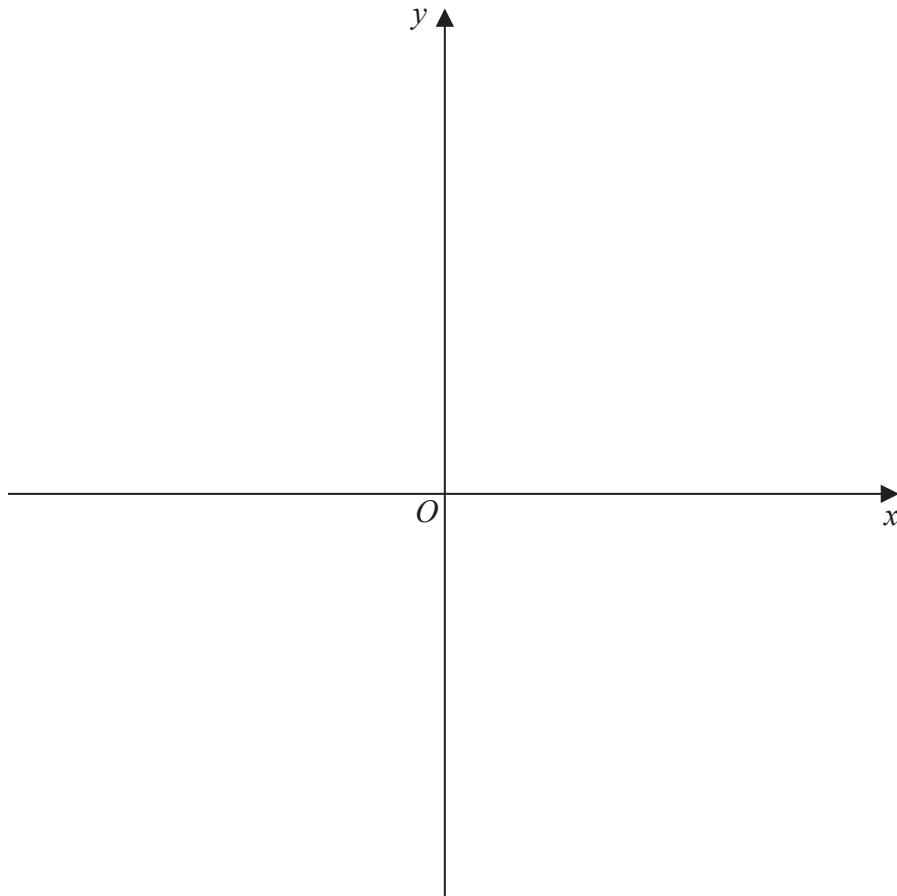
(8)

(e) Hence find the value of

- (i)  $k_1$
- (ii)  $k_2$

(3)



**Question 10 continued**

Turn over for spare axes if you need to redraw your sketch.



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**Question 10 continued**

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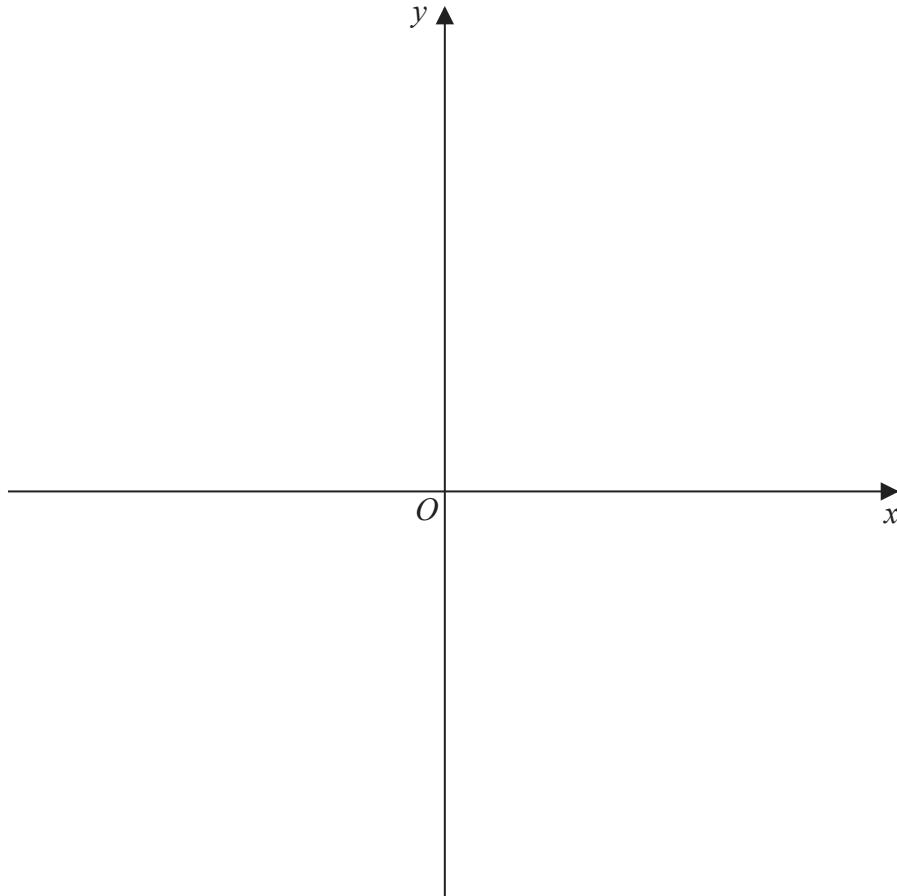
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**Question 10 continued**

**Only use these axes if you need to redraw your sketch.**



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**(Total for Question 10 is 18 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

